COINTEGRATION OF FUNCTIONAL TIME SERIES

U. Deinoravičiūtė, J.Markevičiūtė, P. Treigys

Literature

Integration and Cointegration

A time series which has a unit root is integrated. If two time series are integrated of order I(1) but their linear combination is stationary - the time series are cointegrated.

Functional Time Series

A functional time series $\{X_t(s)\}, t = 1, 2, \ldots, T$ is a series of random functions $X(s), s \in [a, b]$.

FTS Stationarity Tests

- Stationarity in time (Horváth et al., 2014)
- Unit root (Chen & Pun, 2023)
- Stationarity in frequency (Aue & van Delft, 2020)
- Trend stationarity (Kokoszka & Young, 2016)

Functional Time Series Cointegration

Nielsen et al. (2024) developed methods for inferring common stochastic trends and testing hypotheses about stationary and nonstationary subspaces in Hilbert space-valued time series.

A novel method by Seo (2024) offers two advantages: it provides an asymptotically more efficient estimator of cointegrating vectors and enables the development of FPCA-based tests for studying properties of cointegrated FTS.

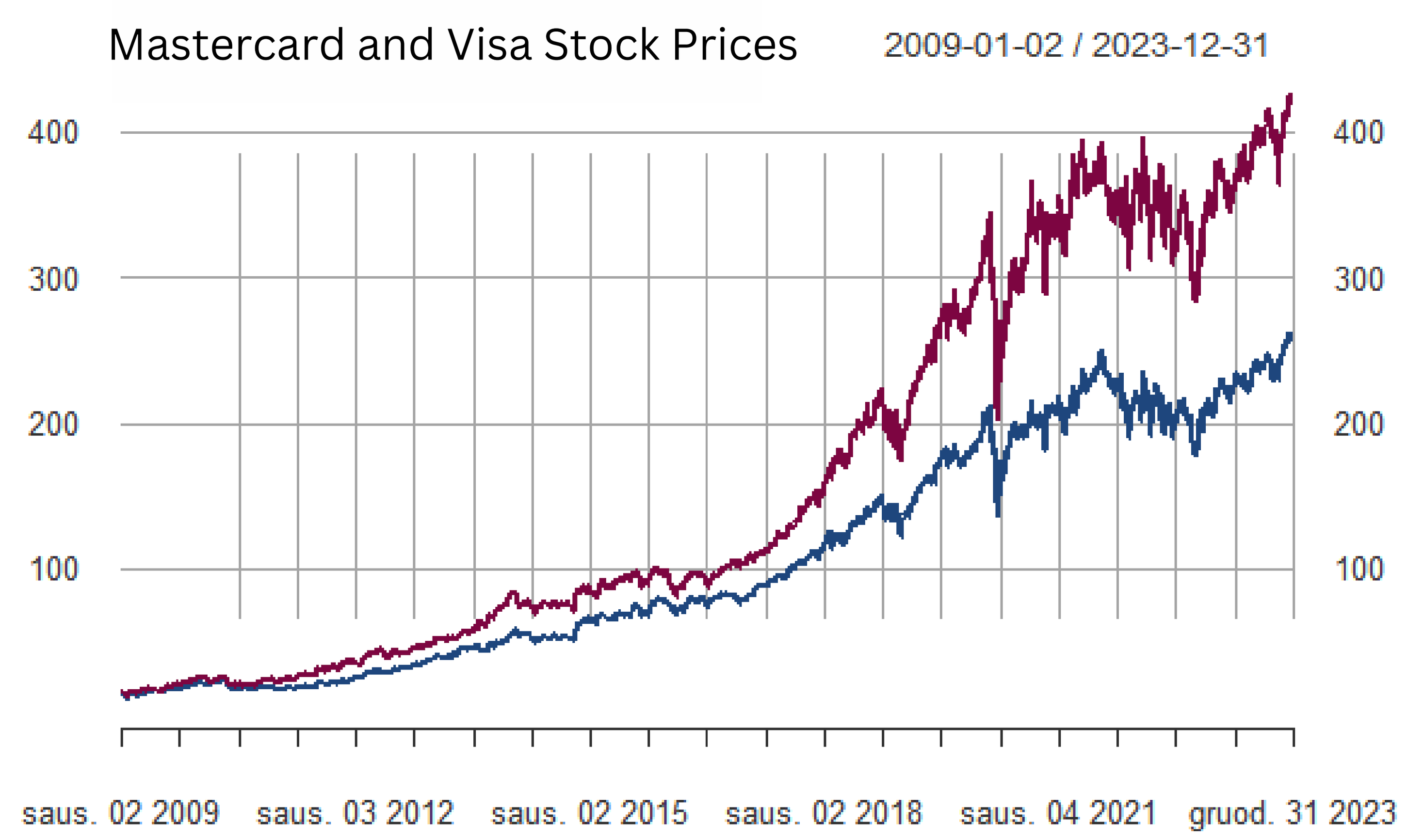
Cointegration Application Examples

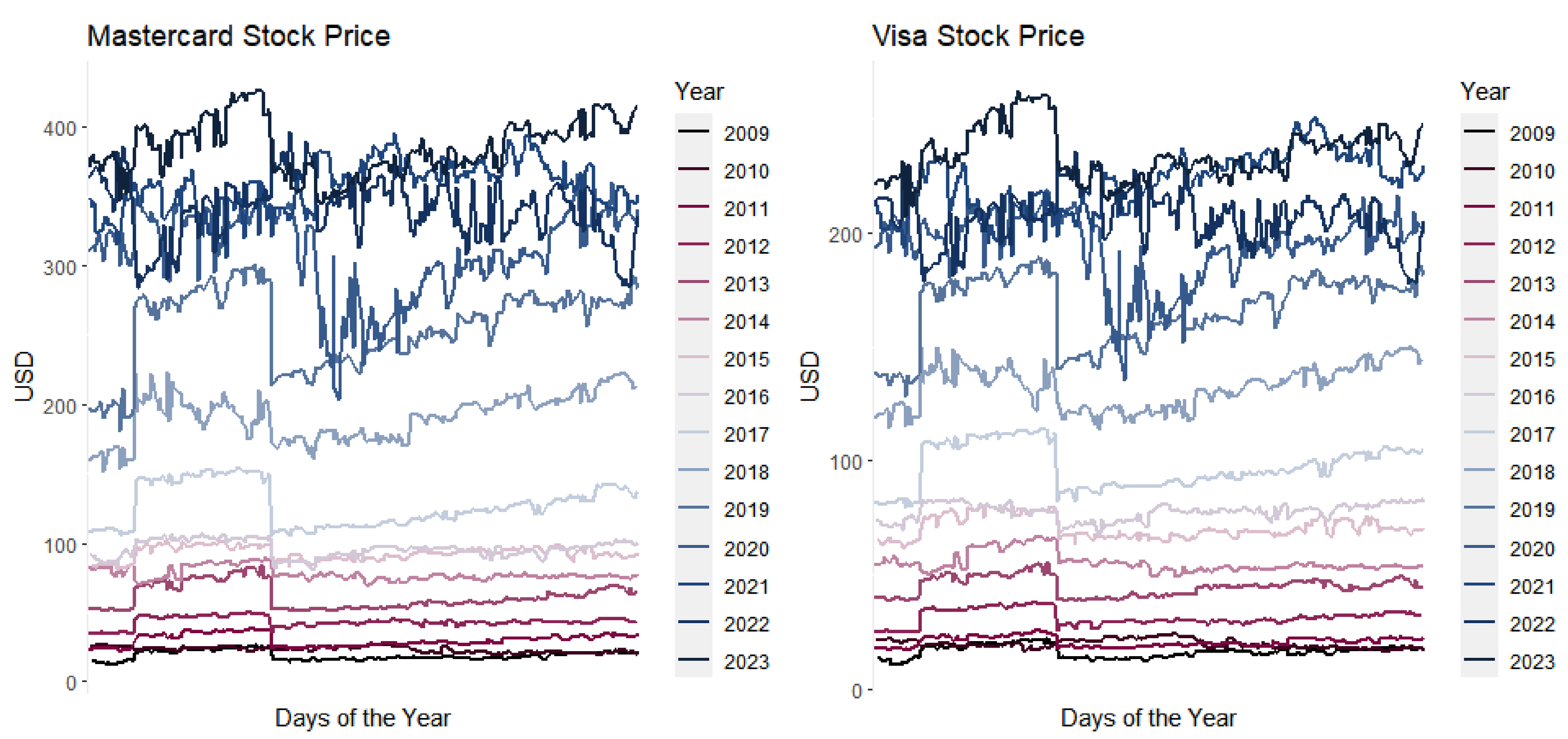
A. E. Maganioti et al., 2010	S. Arnold & V. Glushko, 2021
The study introduced how the concept of cointegration can be useful in analysing ERP brain signals under different electromagnetic field conditions, participants' gender and electrode placement on the brain. The findings indicate that regions on the sides of the brain were more cointegrated and the cointegration factor was higher for women and with EMF present.	The cointegration structure of the underlying causes of death analysed in this study provided insights into their short- and long-term interactions. The findings revealed that, in the long term, the shared trend among cause-specific mortality rates is primarily influenced by cancer, circulatory, and respiratory mortality rates.

Data Analysis

Mastercard / Visa Data

- The dataset (Aslam, 2024) is composed of Mastercard and Visa stock prices from 2009 to 2023, observed daily (close price).
- The data was converted to functional time series where each year was made into a functional observation.
- Analysing simple time series, ADF and KPSS (with and without trend) tests indicated that Mastercard and Visa time series were non-stationary. Conducting these tests on first-differenced data they indicated stationarity, which means that both time series are integrated of order one I(1).
- Johansen test and Engel-Granger two-step method indicated cointegration when performed on simple time series.





Functional KPSS Test		Functional Unit Root Test	
$R_N^{mastercard} = 277.3$	$p^{mastercard} < 2e^{-16}$	$R_N^{mastercard} = 3.9$	$p^{mastercard} = 0.973$
$R_N^{visa} = 106.4$	$p^{visa} < 2e^{-16}$	$R_N^{visa} = 1.2$	$p^{visa} = 0.981$
Both functional time series have a non-stationary trend.		Both functional time series contain a unit root.	
Tests conducted using STFTS package (Chen & Pun, 2021) in R.			

References

- Arnold, S., & Glushko, V. (2021). Short- and Long-Term Dynamics of Cause-Specific Mortality Rates Using Cointegration Analysis. *North American Actuarial Journal*, 26(2), 161–183. https://doi.org/10.1080/10920277.2021.1874421
- Aslam, T. (2024). Stock Prices of MasterCard and Visa (2008-2024) [Data set]. Kaggle. https://www.kaggle.com/datasets/ranatalha71/stock-prices-of-mastercard-and-visa-2008-2024
- Aue, A., & van Delft A. (2020). Testing for stationarity of functional time series in the frequency domain. *The Annals of Statistics* 48(5), 2505-2547. https://doi.org/10.1214/19-AOS1895
- Chen, Y., & Pun, C. S. (2021). An LM-type Unit Root Test for Functional Time Series. http://dx.doi.org/10.2139/ssrn.3761262
- Chen Y., & Pun C. S. (2021). STFTS: Statistical Tests for Functional Time Series [R package].
- Horváth, L., Kokoszka, P., & Rice, G. (2014). Testing stationarity of functional time series. *Journal of Econometrics*, 179(1), 66-82. https://doi.org/10.1016/j.jeconom.2013.11.002.
- Kokoszka, P., & Young, G. (2016). KPSS Test for Functional Time Series. *Statistics* 50, 957-973. https://doi.org/10.1080/02331888.2015.1128937
- Maganioti, A.E., Hountala, C.D., Papageorgiou, C.C., Rabavilas, A.D., Papadimitriou, G.N., Capsalis C.N. (2010). Cointegration of event-related potential (ERP) signals inexperiments with different electromagnetic field (EMF)conditions. *Health* 2(5), 400-406. DOI: 10.4236/health.2010.25060
- Nielsen, M. Ø., Seo, W.-K., & Seong, D. (2024). Inference on common trends in functional time series. https://arxiv.org/abs/2312.00590
- Seo, W.-K. (2024). Functional principal component analysis for cointegrated functional time series. *Journal of Time Series Analysis*, 45, 320-330.

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